VCAA Further Mathematics Exam 1 Solutions 2005

SECTION A Core

1	2	3	4	5	6	7	8	9	10	11	12	13
B	**E**	**D**	**C**	**A**	**C**	**D**	**C**	**B**	**B**	**A**	**B**	**E**

SECTION B

Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
C	**D**	**D**	**C**	**E**	**B**	**A**	**C**	**B**

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
C	**C**	**E**	**B**	**E**	**A**	**B**	**B**	**A**

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
A	**E**	**C**	**C**	**D**	**E**	**B**	**E**	**A**

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
D	**B**	**B**	**A**	**B**	**C**	**A**	**D**	**E**

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
C	**D**	**B**	**B**	**A**	**C**	**D**	**C**	**B**

SECTION A Core

Q1 The speeds are already in order.

The median speed $= \frac{62.8 + 62.6}{2} = 62.7$ km/h B

Q2 7 out of 10, $\therefore \frac{7}{10} \times 100\% = 70\%$ E

Q3 The histogram has outliers and the median is on the left of the mode. D

Q4 3 out of 30, $\therefore \frac{3}{30} \times 100\% = 10\%$ C

Q5 Mean number of mobiles $= \frac{\sum xf}{\sum f}$

$= \frac{0\times 34 + 1\times 78 + 2\times 30 + 3\times 12}{154} = 1.13$ A

Q6 6.6 is one standard deviation less than 8.8. 68% within and 32% outside (less than + more than), $\therefore$ 16% less than 6.6 C

Q7 Smaller interquartile range, $\therefore$ less variable, and higher median, $\therefore$ increase rate of growth. D

Q8 Graphics calculator: $r = 0.9681$ C

Q9 Gradient of equation $= 0.96$ B

Q10 x-coordinates in order: 1, 2, 3, 4, 5 $\therefore$ median x = 3.
y-coordinates in order: 0, 1, 2, 4, 5 $\therefore$ median y = 2.
$\therefore$ median point is $(3,2)$. B

Q11 A seasonal index of 1.1 means 0.1 or $\frac{1}{10}$ above the yearly average. $\therefore \frac{1}{10} \times 100\% = 10\%$. A

Q12 Share price $= 1.24 + 0.06 \times 48 = 4.12$ B

Q13 Positive gradient: increasing linear trend; more fluctuations: increasing variability (volatility). E

Module 1: Number patterns and applications

Q1 Add 6 to get the next number, −5, 1, 7, 13, 19, 25, …. $\therefore$ C

Q2 The first sequence has no common ratio, because $\frac{1.11}{1.1} \neq \frac{1.111}{1.11}$, $\therefore$ not geometric. The rest are geometric. D

Q3 $1200 : x = 3 : 5$, $\frac{1200}{x} = \frac{3}{5}$, $\therefore x = \frac{5\times 1200}{3} = 2000$ D

Q4 The sum of the blocks in the towers forms an arithmetic series: $3+5+7+9+11 = 35$, $3+5+7+9+11+13 = 48$, $3+5+7+9+11+13+15 = 63$. $\therefore$ only six towers following the pattern are possible with 50 blocks. C

Q5 $S_\infty = \frac{a}{1-r} = \frac{80}{1-0.75} = \frac{80}{0.25} = 320$ E

Q6 B

Q7 The terms alternate between positive and negative values with magnitude decreasing exponentially. $\therefore$ the sequence has a negative r and $-1 < r < 0$. A

Q8 An arithmetic sequence has a common difference, i.e. $t_{n+1} - t_n = d$. When compare with $t_{n+1} - at_n = b$, $a = 1$ and b is the common difference that can be any value, say 2. C

Q9 $P_n = 2P_{n-1} - 200$, $\therefore P_{n-1} = \frac{1}{2}(P_n + 200)$.

$\therefore P_5 = \frac{1}{2}(P_6 + 200) = \frac{1}{2}(1000 + 200) = 600$ and

$P_4 = \frac{1}{2}(P_5 + 200) = \frac{1}{2}(600 + 200) = 400$. B

Module 2: Geometry and trigonometry

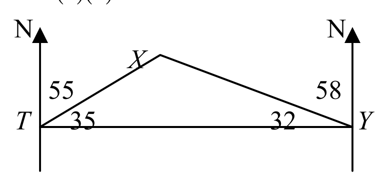
Q1 Area $= \frac{1}{2}(5)(10)\sin 25^o = 10.6$ cm^2. C

Q2 $\angle FOD = 2 \times 60^o = 120^o$ C

Q3 $CB = 2\sqrt{14^2 - 8^2} = 23.0$ m E

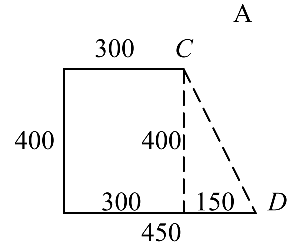
Q4 The cosine rule:
$AB = \sqrt{6^2 + 4^2 - 2(6)(4)\cos 30^o} = 3.2$ km B

Q5



$\angle TXY = 180 - 35 - 32 = 113^o$ E

Q6 $CD = \sqrt{400^2 + 150^2} = 427$ m A



Q7 The sine rule: $\frac{20}{\sin \alpha^o} = \frac{15}{\sin \beta^o}$,

$\therefore \sin \beta^o = \frac{15 \sin \alpha^o}{20} = \frac{15 \times 0.8}{20} = 0.6$ B

Q8 Enlarged area : original area

$= 2250:1440 = 225:144 = 15^2 : 12^2$.

$\therefore$ enlarged length : original length $= 15:12 = x:36$.

$\therefore \frac{x}{36} = \frac{15}{12}$, $x = 45$ m B

Q9 $HM = \sqrt{4^2 + 2^2} = 4.4721$,

$\tan \angle EMH = \frac{4}{4.4721}$, $\angle EMH = 41.8^o$ A

Module 3: Graphs and relations

Q1 A

Q2 Substitute $y = -5$ in $y = -x + 5$, $-5 = -x + 5$,

$\therefore x = 10$. The point is $(10, -5)$ E

Q3 $3x + 5y = 0$, $\therefore y = -\frac{3}{5}x$,

$\therefore$ the line has a negative gradient. C

Q4 Substitute the coordinates of one of the intercepts in the equation, $3(8) + 2(0) = 4k$, $\therefore k = 6$ C

Q5 Charge $= 50 + 65n$ D

Q6 Let x be the price of ice cream and y the price of drink.

$4x + 3y = 21.40$ ……(1)

$5x + 2y = 20.80$ ……(2)

$5 \times (1) - 4 \times (2)$, $7y = 23.80$, $y = 3.40$ E

Q7 Gradient $\frac{4-0}{2-0} = 2$, $\therefore y = 2x$ or $b = 2a^2$. B

Q8 $P = 4x - 3y$, to maximise P, x must be as large as possible and y as small as possible. $\therefore P$ is maximum at $(120, 0)$. E

Q9 $I = \frac{k}{d^2}$, $\therefore Id^2 = k = 20 \times 50^2 = 50000$.

$I = \frac{k}{d^2}$ is not a straight line.

Since the point $(5, 2000)$ on graph A satisfies $Id^2 = 50000$, $\therefore$ A

Module 4: Business-related mathematics

Q1 $r = \frac{100 \times SI}{PT} = \frac{100 \times 140.38}{1200 \times 1} = 11.70$ D

Q2 $110\% \times x = 22$, $1.10x = 22$, $x = 20.00$ B

Q3

Withdrawals	Deposits	Balance
		2143.50
	2.45	2145.95
616.40		1529.55
	x	1971.75

$1529.55 + x = 1971.75$, $\therefore x = 442.20$ B

Q4 Total number of boxes in two years $= 2 \times 120000 = 240000$

Depreciated value $= 45000 - 0.05 \times 240000 = 33000$ A

Q5 $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, where $A = 200000$,

$P = 200000$, $n = 60$, $R = 1 + \frac{8.5}{12 \times 100} = 1.0070833$.

$Q = 1416.67$ B

Q6 Total at end of 3rd year $= PR^n = 3000(1.065)^3 = 3623.85$

Total at end of 4th year $= 3000(1.065)^4 = 3859.40$

$\therefore$ Interest in 4th year $= 3859.40 - 3623.85 = 235.55$ C

Q7 $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, where $P = 10000$, $n = 16$, $Q = 500$,

$R = 1 + \frac{6}{4 \times 100} = 1.015$. $\therefore A = 3723.67$ A

Q8 Let P be the Monday's price.

Friday's price $= P(110\%)(90\%)(120\%)(80\%)$

$= P(1.1)(0.9)(1.2)(0.8) = 0.95P = 95\%P$, $\therefore$ 5% lower than Monday's price. D

Q9 She has to pay interest on the missed third payment. E

Module 5: Networks and decision mathematics

Q1 $1 + 1 + 2 + 4 = 8$ C

Q2 D

Q3 B

Q4 Check by adding. B

Q5 Euler's formula: $V = E - F + 2$,

$10 = E - 3 + 2$, $\therefore E = 11$.

Number of edges removed $= 15 - 11 = 4$. A

Q6 Maximum flow from X to Y = minimum cut.

Cut A $= 2 + 3 + 5 + 5 + 7 = 22$

Cut B is a wrong cut.

Cut C $= 6 + 5 + 7 = 18$

Cut D is a wrong cut.

Cut E $= 6 + 18 = 24$. Cut C is a minimum. C

Q7 D because there are 3 ways from L to M without passing through the other towns, and only one way from M back to M without passing the other towns.

Q8 Find the critical path (the longest path) first.

$PSVXZ = 4 + 3 + 3 + 8 + 6 = 24$

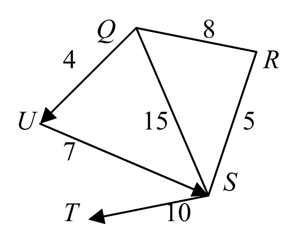
$QTXZ = 5 + 6 + 8 + 6 = 25$

$RUXZ = 12 + 0 + 8 + 6 = 26$

$RWYZ = 12 + 4 + 3 + 6 = 25$

$\therefore RUXZ$ is the critical path. A delay in activity R will cause a delay in completion of the whole project. C

Q9



The shortest route between Q and T is

$QUST = 4 + 7 + 10 = 21$ B

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors